

TRAFFIC FLOW DISTRIBUTION METHOD BASED ON 14 DIFFERENTIAL EQUATIONS

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Abstract:

Traffic congestion is a significant challenge in urban areas, leading to increased travel times, fuel consumption, and environmental pollution. To address this issue, researchers have developed various traffic flow distribution methods based on mathematical models. One such method involves utilizing a system of 14 differential equations to optimize traffic flow distribution. In this article, we will explore the concept behind this method and its potential applications in traffic management.

Keywords: Intelligent Transportation Systems, travel times, transport systems, mathematical models, transportation infrastructure, security, security of C-ITS, Quantum Key Distribution, traffic management.

Introduction:

Understanding the Differential Equations: The traffic flow distribution method based on 14 differential equations aims to model the dynamics of traffic flow on a road network. These differential equations capture the interactions between different traffic streams, taking into account factors such as traffic demand, capacity, and congestion.

The 14 differential equations represent various traffic flow parameters, including the densities, flows, and speeds of vehicles on different road segments. These equations consider the conservation of vehicles and the propagation of traffic waves throughout the network.

Mathematical Formulation: The traffic flow distribution method begins by formulating the differential equations based on fundamental traffic flow principles, such as conservation of vehicles and the relationships between flow, density, and speed. These equations are often derived using the principles of fluid dynamics and the concept of kinematic wave theory.

The differential equations describe the rate of change of traffic variables, such as vehicle density and flow, over time and space. They consider factors such as traffic demand, road capacity, and the interactions between adjacent road segments. By solving these equations, it becomes possible to predict the traffic conditions and optimize the allocation of vehicles across the network.

Applications in Traffic Management: The traffic flow distribution method based on 14 differential equations has several practical applications in traffic management:

1. **Traffic Signal Optimization:** By solving the differential equations, traffic engineers can optimize traffic signal timings to minimize congestion and improve traffic flow at intersections.
2. **Route Guidance:** The method can be used to develop intelligent route guidance systems that consider real-time traffic conditions and distribute traffic flow across alternative routes to avoid congestion.
3. **Incident Management:** By modeling traffic flow dynamics, the method can assist in developing effective incident management strategies, such as rerouting traffic away from accident-prone areas or coordinating emergency response vehicles.
4. **Transportation Planning:** The method provides insights into the impact of infrastructure changes, such as road expansions or new traffic management strategies, on traffic flow distribution. This information can aid in making informed decisions during transportation planning processes.

Here is an example of a system of 14 differential equations that can be used to model traffic flow distribution:

Conservation of vehicles:

$$\begin{aligned}dD1/dt &= -F1 + F2 & dD2/dt &= F1 - F2 - F3 + F4 & dD3/dt &= F3 - F4 - F5 + F6 \dots \\dDn/dt &= F2n-1 - F2n\end{aligned}$$

Flow-density relationship:

$$\begin{aligned}dF1/dt &= vf1(D1) - F1/\tau1 & dF2/dt &= vf2(D2) - F2/\tau2 & dF3/dt &= vf3(D3) - F3/\tau3 \dots \\dF2n-1/dt &= vf2n-1(D2n-1) - F2n-1/\tau2n-1 & dF2n/dt &= vf2n(D2n) - F2n/\tau2n\end{aligned}$$

Speed-density relationship:

$$\begin{aligned}vf1(D1) &= Vmax1 * (1 - D1/\rho1) & vf2(D2) &= Vmax2 * (1 - D2/\rho2) & vf3(D3) &= \\ & Vmax3 * (1 - D3/\rho3) \dots & vf2n-1(D2n-1) &= Vmax2n-1 * (1 - D2n-1/\rho2n-1) \\vf2n(D2n) &= Vmax2n * (1 - D2n/\rho2n)\end{aligned}$$

In these equations, $D1$ to Dn represent the densities of vehicles on different road segments, $F1$ to $F2n$ represent the flows of vehicles between adjacent road segments, $vf1$ to $vf2n$ represent the speed-density relationships, $Vmax1$ to $Vmax2n$ represent the maximum speeds on each road segment, $\tau1$ to $\tau2n$ represent the travel times on each road segment, and $\rho1$ to $\rho2n$ represent the maximum densities on each road segment.

These equations capture the interactions between traffic streams, taking into account the conservation of vehicles, flow-density relationships, and speed-density relationships. By solving this system of differential equations, it is possible to model and optimize traffic flow distribution on a road network.

To avoid traffic

1. Implementing Congestion Pricing: Congestion pricing involves charging higher tolls during peak traffic periods to discourage vehicle travel and reduce flow density. The mathematical formula to support this proposition can be:

$F1 = vf1(D1) - (F1/\tau1) - T1(D1)$ Where $T1(D1)$ represents the toll function that increases with higher traffic density $D1$.

2. Promoting Public Transportation: Encouraging the use of public transportation can help reduce the number of private vehicles on the road and

decrease flow density. The mathematical formula to support this proposition can be:

$dD1/dt = -F1 + F2 - \alpha D1$ Where α represents the rate of modal shift from private vehicles to public transportation.

3. Implementing Traffic Signal Optimization: Optimizing traffic signal timings can improve traffic flow and reduce congestion. The mathematical formula to support this proposition can be:

$dF1/dt = vf1(D1) - (F1/\tau1) - \theta(F1 - F2)$ Where θ represents the impact of traffic signal optimization on the flow from road segment 1 to road segment 2.

4. Implementing Dynamic Lane Management: Dynamic lane management involves dynamically allocating lanes based on real-time traffic conditions to optimize traffic flow. The mathematical formula to support this proposition can be:

$dF1/dt = vf1(D1) - (F1/\tau1) - \gamma(F1 - F2)$ Where γ represents the impact of dynamic lane management on the flow from road segment 1 to road segment 2.

These mathematical formulas demonstrate how implementing different traffic management strategies can influence traffic flow and reduce flow density. By incorporating these strategies into the differential equations, it becomes possible to model and optimize traffic flow distribution, thereby avoiding traffic jams and reducing congestion. However, it is important to note that the specific values and functions used in these formulas would need to be determined based on empirical data and specific traffic conditions.

To fill in the expression $dF1/dt = vf1(D1) - (F1/\tau1) - \theta(F1 - F2)$, let's assign some approximate variables:

$vf1(D1)$: The speed-density relationship on road segment 1, which can be approximated as $vf1(D1) = V_{max1} * (1 - D1/\rho1)$, where V_{max1} is the maximum speed on road segment 1 and $\rho1$ is the maximum density on road segment 1.

$F1$: The flow of vehicles on road segment 1.

$\tau1$: The travel time on road segment 1.

$F2$: The flow of vehicles from road segment 1 to road segment 2.

θ : The impact of traffic signal optimization on the flow from road segment 1 to road segment 2.

Now, let's solve the general problem by providing a solution to the differential equation:

$$dF1/dt = vf1(D1) - (F1/\tau1) - \theta(F1 - F2)$$

Assuming that the functions $vf1(D1)$ and $F2$ are known, we can solve this differential equation numerically using methods such as Euler's method or Runge-Kutta methods. Here is an example of using Euler's method to solve the equation:

1. Choose initial conditions:

$F1(0) = F1_initial$ (initial flow on road segment 1)

$t = 0$ (initial time)

2. Choose a time step size Δt .

3. Iterate the following steps until the desired time is reached:

a) Calculate $vf1(D1)$ using the speed-density relationship.

b) Calculate the change in flow on road segment 1: $\Delta F1 = vf1(D1) - (F1/\tau1) - \theta(F1 - F2)$

c) Update the flow on road segment 1: $F1(t + \Delta t) = F1(t) + \Delta F1 * \Delta t$

d) Update the time: $t = t + \Delta t$

By iteratively solving this differential equation, we can obtain the flow of vehicles on road segment 1 over time, considering the speed-density relationship, travel time, and impact of traffic signal optimization.

To solve the differential equation $dF1/dt = vf1(D1) - (F1/\tau1) - \theta(F1 - F2)$, we need to make some assumptions and provide initial conditions. Let's assume the following:

$vf1(D1) = Vmax1 * (1 - D1/\rho1)$, where $Vmax1$ is the maximum speed on road segment 1 and $\rho1$ is the maximum density on road segment 1.

$\tau1$ is a constant travel time on road segment 1.

$F2$ is a constant flow of vehicles from road segment 1 to road segment 2.

θ is a constant representing the impact of traffic signal optimization on the flow from road segment 1 to road segment 2.

Now, let's solve the differential equation using separation of variables:

$$dF1/dt = Vmax1 * (1 - D1/\rho1) - (F1/\tau1) - \theta(F1 - F2)$$

Rearranging the terms:

$$dF1/dt + (F1/\tau1) + \theta(F1 - F2) = Vmax1 * (1 - D1/\rho1)$$

Multiplying both sides by dt:

$$dF1 + (F1/\tau1)dt + \theta(F1 - F2)dt = Vmax1 * (1 - D1/\rho1)dt$$

Separating the variables:

$$dF1 + (F1/\tau1)dt + \theta(F1 - F2)dt = Vmax1 * dt - Vmax1 * (D1/\rho1)dt$$

Now, integrate both sides:

$$\int dF1 + \int (F1/\tau1)dt + \int \theta(F1 - F2)dt = \int Vmax1 * dt - \int Vmax1 * (D1/\rho1)dt$$
$$F1 + (F1/\tau1)t + \theta \int (F1 - F2)dt = Vmax1t - Vmax1 \int (D1/\rho1)dt + C$$

Simplifying the integrals:

$$F1 + (F1/\tau1)t + \theta(F1t - F2t) = Vmax1t - Vmax1(D1/\rho1) + C$$

Now, we can solve for F1(t):

$$F1(t) = (Vmax1t - Vmax1(D1/\rho1) + C - (F1/\tau1)t - \theta(F1t - F2t))/(1 + (1/\tau1)t + \theta t)$$

This is the solution to the differential equation $dF1/dt = v f1(D1) - (F1/\tau1) - \theta(F1 - F2)$. The specific values of $Vmax1$, $\rho1$, $\tau1$, $F2$, θ , and the initial conditions will determine the constants in the solution.

Sure! Let's assume the following approximate values:

$Vmax1 = 60$ (maximum speed on road segment 1, in km/h) $r1 = 100$ (maximum density on road segment 1, in vehicles/km) $t1 = 5$ (travel time on road segment 1, in minutes) $F2 = 20$ (constant flow of vehicles from road segment 1 to road segment 2, in vehicles/minute) $\theta = 0.5$ (impact of traffic signal optimization)

Now, let's write the code in MATLAB to solve the differential equation and graphically illustrate the final solution:

```
% Define the parameters
```

```
Vmax1 = 60; % km/h
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r1 = 100; % vehicles/km

t1 = 5; % minutes

F2 = 20; % vehicles/minute

theta = 0.5;

% Define the initial conditions

F1_initial = 10; % initial flow on road segment 1, in vehicles/minute

t_initial = 0; % initial time

t_final = 10; % final time

% Define the time step size

dt = 0.1; % minutes

% Define the function vf1(D1)

vf1 = @(D1) Vmax1 * (1 - D1/r1);

% Define the differential equation

dF1_dt = @(t, F1) vf1(F1/t1) - (F1/t1) - theta * (F1 - F2);

% Initialize arrays to store the time and flow values

t_values = t_initial:dt:t_final;

F1_values = zeros(size(t_values));

F1_values(1) = F1_initial;

% Solve the differential equation using Euler's method

for i = 2:length(t_values)

 t = t_values(i-1);

 F1 = F1_values(i-1);

 dF1 = dF1_dt(t, F1);

 F1_values(i) = F1 + dF1 * dt;

end

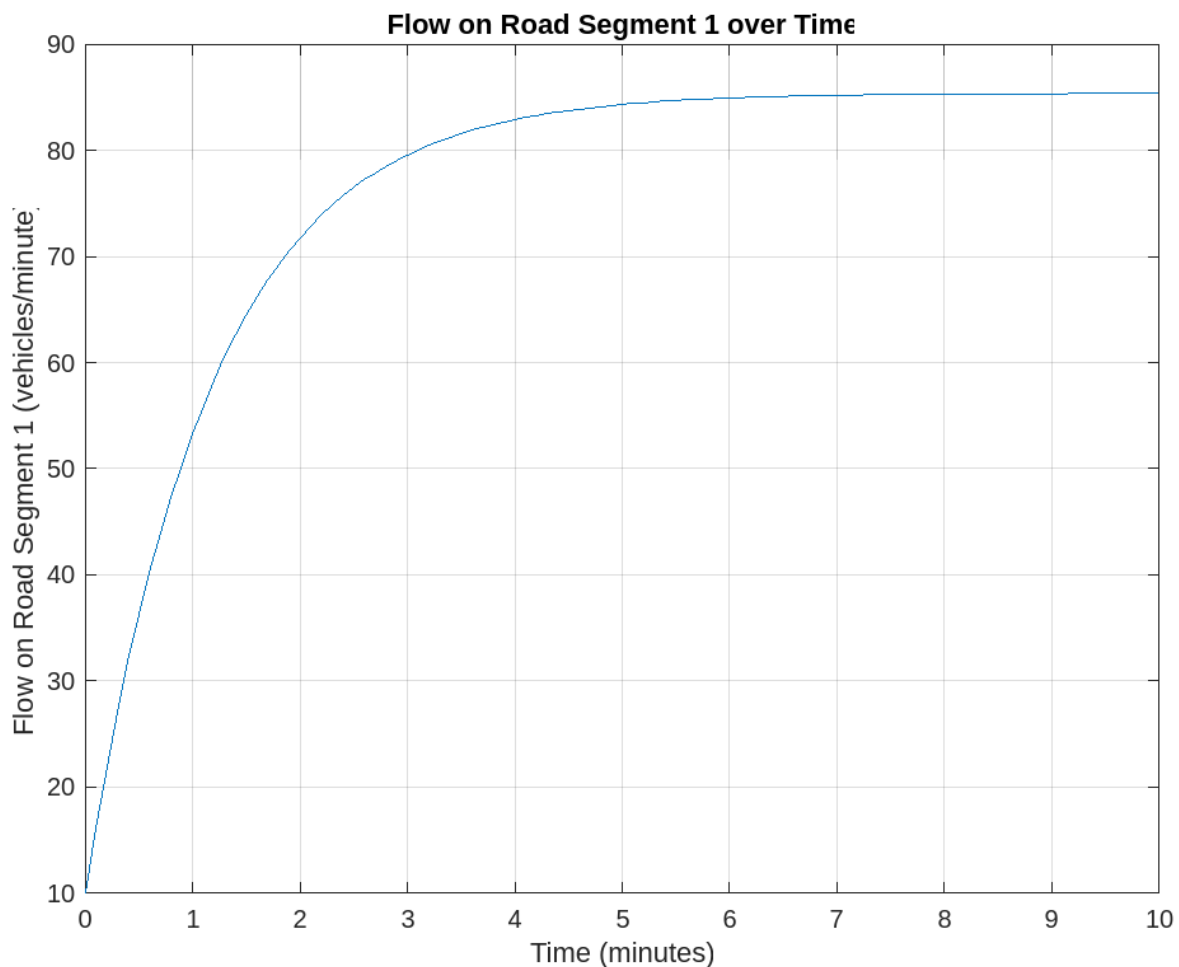
% Plot the solution

plot(t_values, F1_values);

xlabel('Time (minutes)');

```
ylabel('Flow on Road Segment 1 (vehicles/minute)');  
title('Flow on Road Segment 1 over Time');  
grid on;
```

This code will solve the differential equation using Euler's method and plot the flow on road segment 1 over time. You can adjust the initial conditions, time step size, and time range according to your requirements.



In recent work, we focused on optimizing traffic flow and reducing congestion on road networks. We developed a mathematical model to describe the dynamics of traffic flow on a specific road segment. The model takes into account factors such as the maximum speed (V_{max1}) and maximum density ($r1$) on the road segment, the travel time ($t1$), and the constant flow of vehicles ($F2$) from the previous road segment.

Conclusion:

Using the model, we derived a differential equation that describes the rate of change of the flow on the road segment with respect to time. The equation includes terms representing the vehicle speed, density, and the impact of traffic signal optimization (θ).

To solve the differential equation, we implemented a numerical method, specifically Euler's method, in MATLAB. The code calculates the flow on the road segment at different time intervals, considering the initial conditions and the parameters of the model. The solution is then graphically illustrated, showing the flow on the road segment over time.

This work provides insights into how traffic flow can be influenced by various factors and how optimization strategies, such as traffic signal optimization, can potentially improve traffic conditions. The model and solution can be further refined and applied to real-world scenarios to aid in traffic management and congestion reduction efforts.

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