Intent Research Scientific Journal-(IRSJ)
ISSN (E): 2980-4612
Volume 2, Issue 6, June -2023
Website: intentresearch.org/index.php/irsj/index

ANIQ INTEGRALLARNI TAQRIBIY HISOBLASH

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Annotatsiya:
Matematik analiz kursidan ma'lumki, agar integral ostidagi funksiya murakkab bo'lsa, tegishli aniq integralni hisoblashni taqribiy usullarini qo'llash lozim bo'ladi. Ushbu maqolada aniq integrallarni taqribiy hisoblashga oid tushunchalar va misollar ko'rib chiqilgan.

Kalit so'z: taqribiy hisoblash usullari, to'g'ri tortburchak usuli, trapetsiya usuli, simpson formulasi, bo'laklarga bo'lish, bo'lak uzunliklari.

Аннотация:
Если функция под интегралом сложна, необходимо использовать приближенные методы вычисления соответствующего точного интеграла. В этой статье рассматриваются концепции и примеры приближенного вычисления определенных интегралов.

Ключевое слово: методы приближенного вычисления, метод прямоугольного треугольника, метод трапеции, формула Симпсона, деление на части, длины частей.

Annotation:
But the question of finding the initial function is not always easily solved. If the function under the integral is complex, it becomes necessary to apply approximate methods of calculating the corresponding exact integral. This article examines concepts and examples of approximate computation of concrete integrals.

Keyword: approximate calculation methods, correct tortoise method, trapezoidal method, simpson formula, partitioning, slice lengths.

Matematik analiz kursidan bilamizki, integral ostidagi funksiyaning boshlang‘ich funksiyasi ma’lum bo‘lsa integralni Nyuton-Leybnits formulasi yordamida hisoblash mumkin [6]. Misol sifatida quyidagicha bir nechta misollarni keltirib o’tamiz.

Masalan: $\int_1^3 5x^2dx = \frac{5}{3}x^3\Big|_1^3 = \frac{5}{3}\cdot 3^3 - \frac{5}{3}\cdot 1^3 = \frac{135}{3} - \frac{5}{3} = 43\frac{1}{3}$

Ammo boshlang‘ich funksiyani topish masalasi doim osongina hal bo‘lavermaydi. Masalan e^{-x^2} $\frac{1}{\lg x}$ $\frac{1}{1+x}$ $e^{\sin x}$ kabi misollarni yechishni tahlil qilamiz. Buning uchun quyidagi formulalarni va xossalarni o’rganib chiqamiz.

1. To‘g‘ri to‘rtburchaklar formulasi

$f(x)$ funksiya $[a,\ b]$ segmentda berilgan va uzluksiz bo‘lsin. Bu funksiyaning aniq integrali

$$\int_a^b f(x)dx \text{ ni}$$

taqribiy ifodalovchi formulani keltiramiz, $[a,\ b]$ segmentni

$$a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$$

nuqtalar yordamida n ta teng bo‘lakka bo‘lamiz.

Bu holda

$$\Delta x_k = x_{k+1} - x_k = \frac{b-a}{n}, \qquad x_k = a + k\frac{b-a}{n}, \quad (k = 0,1,\dots,n)$$

bo‘ladi.

Berilgan $f(x)$ funksiyaning x_k nuqtadagi qiymati $f(x_k)$ ni hisoblab, $f(x)$ funksiyaning $[x_k,\ x_{k+1}]$ segment bo‘yicha aniq integralini quyidagicha

$$\int_{x_k}^{x_{k+1}} f(x)dx \approx f(x_k)\cdot\Delta x_k = f(x_k)\cdot\frac{b-a}{n}$$

taqribiy ifodalaymiz. Bunday taqribiy formulani har bir $[x_k,\ x_{k+1}]$, $(k = 0, 1, 2, \dots\dots, n-1)$ segmentga nisbatan yozib, so‘ng ularni hadlab qo‘shib topamiz:

$$\int_{a}^{x_1} f(x)dx \approx f(x_0)\cdot\frac{b-a}{n},$$

$$\int_{x_1}^{x_2} f(x)dx \approx f(x_1)\cdot\frac{b-a}{n},$$

$$\int_{x_2}^{x_3} f(x)dx \approx f(x_2)\cdot\frac{b-a}{n}$$

$$.\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .$$

$$\int_{x_{n-1}}^{b} f(x)dx \approx f(x_{n-1})\cdot\frac{b-a}{n},$$

$$\int_{a}^{x_1} f(x)dx + \int_{x_1}^{x_2} f(x)dx + \int_{x_2}^{x_3} f(x)dx + \cdots + \int_{x_{n-1}}^{b} f(x)dx \approx$$

$$\approx \frac{b-a}{n}[f(x_0)+f(x_1)+f(x_2)+\cdots+f(x_{n-1})].$$

Demak,

$$\int_{a}^{b} f(x)dx \approx \frac{b-a}{n}[f(x_0)+f(x_1)+f(x_2)+\cdots+f(x_{n-1})],$$

$$\int_{a}^{b} f(x)dx \approx \frac{b-a}{n}\sum_{k=0}^{n-1} f(x_k). \qquad (1)$$

Bu aniq integralni taqribiy hisoblovchi formula to‘g‘ri to‘rtburchaklar formulasi deyiladi.[1]

2. Trapetsiyalar formulasi

$f(x)$ funksiya $[a,\ b]$ segmentda berilgan va uzluksiz bo‘lsin, $[a,\ b]$ segmentni yuqoridagidek n ta teng bo‘lakka bo‘lib, $f(x)$ funksiyaning $[x_k,\ x_{k+1}]$ segment bo‘yicha olingan aniq integralini quyidagicha

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$$\int_{x_k}^{x_{k+1}} f(x)dx \approx \frac{f(x_k)+f(x_{k+1})}{2}\cdot \Delta x_k = \frac{f(x_k)+f(x_{k+1})}{2}\cdot\frac{b-a}{n}$$

taqribiy ifodalaymiz. Bunday taqribiy formulani har bir $[x_k,\ x_{k+1}]$, $(k=0,1,2,\dots\dots,n-1)$ segmentga nisbatan yozib, so'ng ularni hadlab qo'shib topamiz:

$$\int_a^{x_1} f(x)dx + \int_{x_1}^{x_2} f(x)dx + \int_{x_2}^{x_3} f(x)dx + \cdots + \int_{x_{n-1}}^{b} f(x)dx \approx$$

$$\approx \frac{b-a}{2n}[(f(x_0)+f(x_1)+\big(f(x_1)+f(x_2)\big)+\big(f(x_2)+f(x_3)\big)+$$

$$+\cdots+(f(x_{n-1})+f(x_n))] =$$

$$= \frac{b-a}{2n}[f(x_0)+2f(x_1)+2f(x_2)+\cdots+2f(x_{n-1})+f(x_n)].$$

Demak,

$$\int_a^b f(x)dx \approx \frac{b-a}{n}\left[\frac{f(x_0)+f(x_n)}{2}+f(x_1+f(x_2)+\cdots+f(x_{n-1})\right]. \qquad (2)$$

Bu aniq integralni taqribiy hisoblovchi formulaga trapetsiyalar formulasi deyiladi.[2]

3. Parabolalar (Simpson) Formulasi

$f(x)$ funksiya $[a,\ b]$ segmentda berilgan va uzluksiz bo'lsin. $[a,\ b]$ segmentni

$$a = x_0 < x_1 < x_2 < \cdots < x_{2n-2} < x_{2n-1} < x_{2n} = b$$

nuqtalar yordamida $2n$ ta teng bo'lakka bo'lamiz.

$f(x)$ funksiyaning $[x_{2k},\ x_{2k+2}]$ segment bo'yicha aniq integralini quyidagicha

$$\int_{x_{2k}}^{x_{2k+2}} f(x)dx \approx \frac{b-a}{6}[f(x_{2k})+4f(x_{2k+1})+f(x_{2k+2})]$$

taqribiy ifodalaymiz. Bu taqribiy formulani har bir

$$[x_{2k}, x_{2k+2}] \qquad (k=0,1,2,\dots\dots,n-1)$$

segmentga nisbatan yozib, so'ng ularni hadlab qo'shib topamiz:

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$$\int_{x_0}^{x_2} f(x)dx + \int_{x_2}^{x_4} f(x)dx + \int_{x_4}^{x_6} f(x)dx + \cdots + \int_{x_{2n-2}}^{x_{2n}} f(x)dx \approx$$

$$\approx \frac{b-a}{6n}\left[\left(f(x_0) + 4f(x_1) + f(x_2)\right) + \left(f(x_2) + 4f(x_3) + f(x_4)\right) + \right.$$

$$\left. +\left(f(x_4) + 4f(x_5) + f(x_6)\right) + \cdots \left(f(x_{2n-2}) + 4f(x_{2n-1}) + f(x_{2n})\right)\right] =$$

$$= \frac{b-a}{6n}\left[\left(f(x_0) + f(x_{2n})\right) + 4\left(f(x_1) + f(x_3) + f(x_5) + \cdots + f(x_{2n-1})\right) + \right.$$

$$\left. +2\left(f(x_2) + f(x_4) + f(x_6)\right) + \cdots + f(x_{2n-2})\right)\right].$$

Demak,

$$\int_a^b f(x)dx \approx$$

$$\approx \frac{b-a}{6n}\left[\left(f(x_0) + f(x_{2n}) + 4\left(f(x_1) + f(x_3) + \cdots + f(x_{2n-1})\right) + \right.\right.$$

$$\left.\left. +2\left(f(x_2) + f(x_4) + \cdots + f(x_{2n-2})\right)\right)\right]. \qquad (3)$$

Bu (3) formula parabolalar (Simpson) formulasi deyiladi.[1,2,3]

Biz yuqorida o'rgangan formulalarimiz orqali ba'zi misollar yordamida yanada chuqurroq o'rganib olamiz.

Misol. Ushbu

$$\int_0^1 e^{-x^2} dx$$

aniq integral to'g'ri to'rtburchaklar, trapetsiyalar va Simpson formulalari yordamida taqribiy hisoblansin.[5]

$\lhd$ [0,1] segmentni 5 ta teng bo'lakka bo'lamiz:

$$0 = x_0; \quad x_1 = 0,2; \quad x_2 = 0,4; \quad x_3 = 0,6; \quad x_4 = 0,8; \quad x_5 = 1.$$

Bu nuqtalarda $f(x) = e^{-x^2}$ funksiyaning qiymatlari quyidagicha bo'ladi:

$$f(x_0) = 1,00000,$$
$$f(x_1) = 0,96079,$$
$$f(x_2) = 0,85214,$$
$$f(x_3) = 0,69768,$$
$$f(x_4) = 0,52729,$$
$$f(x_5) = 0,36788.$$

Har bir bo‘lakning o‘rtasini ifodalovchi nuqtalar quyidagicha

$$x_{\frac{1}{2}} = 0{,}1;\quad x_{\frac{3}{2}} = 0{,}3;\quad x_{\frac{5}{2}} = 0{,}5;\quad x_{\frac{7}{2}} = 0{,}7;\quad x_{\frac{9}{2}} = 0{,}9$$

bo‘lib, bu nuqtalardagi qiymatlari esa quyidagicha bo‘ladi:

$$f\left(x_{\frac{1}{2}}\right) = 0{,}99005,$$

$$f\left(x_{\frac{3}{2}}\right) = 0{,}91393,$$

$$f\left(x_{\frac{5}{2}}\right) = 0{,}77680,$$

$$f\left(x_{\frac{7}{2}}\right) = 0{,}61263,$$

$$f\left(x_{\frac{9}{2}}\right) = 0{,}44486.$$

a). To‘g‘ri tortburchaklar formulasi (1) bo‘yicha.

$$\int_0^1 e^{-x^2}dx \approx$$

$$\approx \frac{1}{5}(0{,}99005 + 0{,}91393 + 0{,}77680 + 0{,}61263 + 0{,}44486) =$$

$$= \frac{1}{5}\cdot 3{,}74027 \approx 0{,}74805.$$

b). Trapetsiyalar formulasi (2) bo‘yicha

$$\int_0^1 e^{-x^2}dx \approx \frac{1}{5}(\frac{1{,}00000 + 0{,}36788}{2} + 0{,}96079 + 0{,}85214 +$$

$$+0{,}69768 + 0{,}52729) = \frac{1}{5}(0{,}68394 + 3{,}03790) =$$

$$= \frac{1}{5}\cdot 3{,}72184 \approx 0{,}74437.$$

v) Simpson formulasi (3) bo‘yicha

$$\int_0^1 e^{-x^2}dx \approx \frac{1}{30}[\,(1{,}00000 + 0{,}36788) +$$

$$+4(0{,}99005 + 0{,}91393 + 0{,}77680 + 0{,}61263 + 0{,}44486) +$$

$$+2(0{,}96079 + 0{,}85214 + 0{,}69768 + 0{,}52729)] =$$

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$$\frac{1}{30}(1{,}36788 + 4 \cdot 3{,}74027 + 2 \cdot 3{,}03790) =$$

$$\frac{1}{30}(1{,}36788 + 6{,}07580 + 14{,}96108) \approx 0{,}74682.$$

Taqribiy formulalar yordamida hisoblab topilgan

$$\int_0^1 e^{-x^2} dx$$

integralning qiymatini, uning

$$\int_0^1 e^{-x^2} dx = 0{,}74685 \ldots$$

qiymati bilan taqqoslab, Simpson formulasi yordamida topilgan integralning taqribiy qiymati aniqroq ekanini ko‘ramiz.

g). To‘g‘ri tortburchaklar formulasi (1) bo‘yicha

$$\int_2^{10} \frac{dx}{\lg x} \approx 1 \cdot \left[f(x_0) + f(x_1) + \ldots\ldots\ldots f(x_7) \right] = 1 \cdot 13{,}1432 = 13{,}1432$$

d) Trapetsiya usulida, (2) formula bo’yicha

$$\int_2^{10} \frac{dx}{\lg x} \approx 1 \cdot \left[\frac{f(x_0) + f(x_8)}{2} + f(x_1) + \ldots\ldots\ldots f(x_7) \right] =$$

$$\frac{3{,}3211 + 1}{2} + 9{,}8221 = 2{,}1605 + 9{,}8221 = 11{,}9826$$

Xulosa o’rnida aytish mumkinki, yuqoridagi tahlillardan ko’rinadiki, agar integral ostidagi funksiya murakkab bo‘lsa, tegishli aniq integralni hisoblashni taqribiy usullarini qo‘llash lozim bo‘ladi. Bu usullar orqali talabalarga yanada fanga bo’lgan qiziqishini oshirish mumkin.

Foydalanilgan adabiyotlar ro’yxati

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